Q^2–Dependence of the Gerasimov-Drell-Hearn Sum Rule

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Abstract

We test the Gerasimov-Drell-Hearn (GDH) sum rule numerically by calculating the total photon absorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ on the nucleon via photon excitation of baryon resonances in the constituent quark model. A total of seventeen, low-lying, non-strange baryon resonances are included in this calculation. The transverse and longitudinal interference cross section, $\sigma_{1/2}^{TL}$, is found to play an important role in the study of the Q^2 variation of the sum rule. The results show that the GDH sum rule is saturated by these resonances at a confidence level of 94%. In particular, the $P_{33}(1232)$ excitation largely saturates the sum rule at $Q^2 = 0$, and dominates at small Q^2. The GDH integral has a strong Q^2-dependence below $Q^2 = 1.0$ GeV^2 and changes its sign around $Q^2 = 0.3$ GeV^2. It becomes weakly Q^2-dependent for $Q^2 > 1.0$ GeV^2 because of the quick decline of the resonance contributions. We point out that the Q^2 variation of the GDH sum rule is very important for understanding the nucleon spin structure in the non-perturbative QCD region.

I. INTRODUCTION

The GDH sum rule connects the helicity structure of the total photo-absorption cross section to the ground state properties of the target nucleon. Based on general principles such as Lorentz invariance, electromagnetic gauge invariance, crossing symmetry, causality, unitarity, and the less sound assumption that one can use an unsubtracted dispersion relation for the spin-dependent part of the forward Compton scattering amplitude, the GDH sum rule can be written as [1]

$$\int_{\nu_{thr}}^{\infty} \frac{d\nu}{\nu}[\sigma_{1/2}(\nu, Q^2 = 0) - \sigma_{3/2}(\nu, Q^2 = 0)] = -\frac{2\pi^2\alpha}{M^2}\kappa^2. \tag{1}$$

Here ν is the photon energy in the laboratory frame, Q^2 $(= -q^2)$ is the momentum transfer squared (zero for a real photon), $\sigma_{1/2}$ and $\sigma_{3/2}$ are the total photoabsorption cross sections for the total helicity $1/2$ and $3/2$ cases, M is the nuclon mass, α the hyperfine structure constant and κ the anomalous magnetic moment of the target nucleon. On the other hand, the analysis of the EMC experimental data (polarized muon deep inelastic scattering) suggests [2]

$$\int_{\nu_{thr}}^{\infty} \frac{d\nu}{\nu}[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)] = \frac{0.2}{Q^2}\left(\frac{8\pi^2\alpha}{M^2}\right) \tag{2}$$

in the region around $Q^2 = 10$ GeV^2. A comparison of Eq. (1) and Eq. (2) indicates that satisfying the GDH sum rule requires a dramatic change in the helicity structure of the photon-nucleon coupling between the real photon limit ($Q^2 = 0$) and the deep inelastic region.

This apparent sign change of the sum rule integral signifies a change of reaction mechanism between the deep inelastic scattering regime and the region of baryon resonance excitation at low and medium energies. This sharp Q^2 dependence of the GDH sum rule provides an important consistency check for quark based hadronic models and our deep understanding of the structure of hadrons. The idea that low lying resonances might saturate the GDH sum rule was discussed many years ago [3] and has been investigated more

quantitatively by Burkert and Li [4]. On the experimental side, however, the sum rule has never been directly measured because the technical developments of the necessary polarized beam and target were not available. However there are now a number of new experiments planned or underway at facilities like ELSA [5], GRAAL [6], LEGS [7], and MAMI [8].

Since the sum rule is derived at zero Q^2, the baryon resonance excitation mechanism dominates the photon-nucleon coupling. Of particular interest is the question of whether the sum rule is truly saturated by low-lying baryon resonances. To what extent do they saturate and which resonance contributes the most? Answers to these questions will be very interesting with regard to the interplay of nuclear and particle physics. In this article, we study the GDH sum rule in the constituent quark model. We briefly present the GDH sum rule and our main physics ideas in section 2, followed by the necessary formalism for the helicity amplitudes in section 3. In section 4, we show our results and give a few concluding remarks.

II. THE GENERALIZED GDH SUM RULE

To understand the GDH sum rule, we start with a brief derivation considering the forward Compton scattering amplitude of a real photon on a nucleon. The corresponding scattering amplitude can be written out between the initial and final nucleon Pauli spinor χ_i and χ_f,

$$T(\nu, \theta = 0) = \chi_f^\dagger[\vec{\epsilon}_f^* \cdot \vec{\epsilon}_i f(\nu) + i\vec{\sigma} \cdot (\vec{\epsilon}_f^* \times \vec{\epsilon}_i)g(\nu)]\chi_i, \tag{3}$$

where $f(\nu)$ and $g(\nu)$ are spin non-flip and flip amplitudes, $\vec{\epsilon}_f$ and $\vec{\epsilon}_i$ denote the polarization vectors of the initial and final photon and $\vec{\sigma}$ is the spin of the nucleon. Both amplitudes, $f(\nu)$ and $g(\nu)$, are functions of the photon energy ν and can be expanded into a power series for small ν, where the leading terms are determined by low energy theorems which are based only on Lorentz and gauge invariance [9].

$$f(\nu) = -\frac{e^2}{m} + (\alpha_N + \beta_N)\nu^2 + O(\nu^4), \tag{4}$$

$$g(\nu) = -\frac{e^2\kappa^2}{2m^2}\nu + \gamma_N\nu^3 + O(\nu^5). \tag{5}$$

Here the first term in $f(\nu)$ is the Thomson limit, and the next term is the contribution of the scalar polarizabilities of the nucleon (α_N for electric and β_N for magnetic). Similarly, the leading term of $g(\nu)$ is from the anomalous magnetic moment κ, the next order is the vector polarizability of the nucleon.

The two independent amplitudes, $f(\nu)$ and $g(\nu)$, can be determined by an experiment using circularly polarized photons and a polarized nucleon target with spin parallel ($J_z = 3/2$) and antiparallel ($J_z = 1/2$) to the photon momentum. The corresponding amplitudes may be expressed as

$$T_{3/2} = f - g, \qquad\qquad T_{1/2} = f + g. \qquad\qquad (6)$$

The optical theorem (unitarity) relates the imaginary parts of these amplitudes to the corresponding total photoabsorption cross sections,

$$\text{Im}[T_{1/2,3/2}(\nu)] = \frac{\nu}{4\pi}\sigma_{1/2,3/2}(\nu). \qquad\qquad (7)$$

From crossing symmetry, $f(\nu)$ must be even and $g(\nu)$ odd under the transformation, $\nu \to -\nu$. Thus, on the basis of analyticity, unitarity and crossing symmetry, we can now write a dispersion relation for $g(\nu)$

$$\text{Re}[g(\nu)] = \frac{2\nu}{\pi}\,\text{P}\int_{\nu_{thr}}^{\infty}\frac{d\nu'}{\nu'^2 - \nu^2}\frac{\nu'}{4\pi}\frac{\sigma_{1/2} - \sigma_{3/2}}{2}, \qquad\qquad (8)$$

The use of an unsubtracted dispersion relation relies on the additional, reasonable hypothesis that $|g(\nu)| \to 0$ as $\nu \to \infty$. Since the threshold energy is of the order of the pion mass, this expression may be expanded as a power series in ν as well. Comparing the resulting series with the low energy expansion, Eq. (5), we can easily obtain the GDH sum rule given in Eq.(1). Similarly, by taking the third derivative of $\text{Re}[g(\nu)]$ we obtain the Burkardt-Cottingham (BC) sum rule, which relates the helicity amplitudes to the nucleon vector polarizability [10],

$$\int_{\nu_{thr}}^{\infty}\frac{d\nu}{\nu^3}[\sigma_{1/2}(\nu,0) - \sigma_{3/2}(\nu,0)] = 4\pi^2\gamma_N. \qquad\qquad (9)$$

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In QCD, the GDH sum rule provides an important constraint on the spin structure of the composite nucleon. It is complementary to the important Bjorken [11] and Ellis-Jaffe [12] sum rules which relate the first moment of the nucleon's first spin structure function, $g_1(x, Q^2)$, to the axial charge of the nucleon [13]. It should be emphasized that the GDH sum rule in Eq.(1) is derived for a real photon which is transverse. For electron-nucleon scattering (shown in Fig. 1), however, the exchanged photon must be virtual and hence it can also be longitudinal ($\lambda = 0$). In this case, the additional interference term between transverse and longitudinal photons, $\sigma_{1/2}^{TL}$, also contributes to the Q^2 dependence of the GDH sum rule. Thus, one should not just consider the Q^2 dependence of the difference ($\sigma_{1/2}^{T} - \sigma_{3/2}^{T}$) alone in studying the Q^2 dependence of the GDH sum rule.

As is well known, the double differential cross section for electron deep inelastic scattering from a nucleon in one photon exchange approximation (see Fig. 1 for notation and kinematics) can be written as the scalar product of a leptonic tensor $L_{\mu\nu}$ and a hadronic tensor $W_{\mu\nu}$ [14]

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}, \tag{10}$$

with $Q^2 = -(k_\mu - k'_\mu)^2$. Both tensors can be decomposed into a symmetric (S) and an antisymmetic part (A), so that

$$L_{\mu\nu} = L_{\mu\nu}^{(S)} + i L_{\mu\nu}^{(A)}, \tag{11}$$

$$W_{\mu\nu} = W_{\mu\nu}^{(S)} + i W_{\mu\nu}^{(A)}, \tag{12}$$

where the $L_{\mu\nu}^{(S)}$ and $L_{\mu\nu}^{(A)}$ are completely known and are given by

$$L_{\mu\nu}^{(S)} = 2[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k \cdot k' - m_e^2)], \tag{13}$$

$$L_{\mu\nu}^{(A)} = 2\epsilon^{\mu\nu\lambda\sigma} q_\lambda \sigma_\sigma, \tag{14}$$

with σ being the lepton polarization vector and $g_{\mu\nu}$ the metric tensor. The unknown $W_{\mu\nu}$, which describes the internal structure of the nucleon, depends on non-perturbative QCD dynamics. In principle, it can be expressed as follows,

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$$W_{\mu\nu}^{(S)} = (\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})W_1(\nu, Q^2) + \frac{1}{M^2}(p_\mu - \frac{p \cdot q}{q^2}q_\mu)(p_\nu - \frac{p \cdot q}{q^2}q_\nu)W_2(\nu, Q^2), \tag{15}$$

$$W_{\mu\nu}^{(A)} = \epsilon_{\mu\nu\lambda\sigma}q^\lambda\{s^\sigma M[G_1(\nu, Q^2) + \frac{p \cdot q}{M^2}G_2(\nu, Q^2)] - p^\sigma\frac{s \cdot q}{M}G_2(\nu, Q^2)\}, \tag{16}$$

where s is the nucleon polarization vector, the $W_{1,2}(\nu, Q^)$ and $G_{1,2}(\nu, Q^2)$ refer to unpolarized and polarized structure functions (scalar functions), respectively. The functions $W_{1,2}(\nu, Q^2)$, which have played a seminal role in the development of our current understanding of hadron structure, carry information about the overall distribution (density) of quarks and gluons in the nucleon. The function $G_1(\nu, Q^2)$ probes the spin distribution of quarks in a polarized nucleon, while $G_2(\nu, Q^2)$ involves higher twist contributions and its physical interpretation is obscure.

The total virtual photoabsorption cross section can then be expressed in terms of the hadronic tensor $W_{\mu\nu}$ [15]

$$\sigma_\lambda^{\gamma^*N} = \frac{4\pi^2\alpha}{K}\epsilon^{\mu*}(\lambda)W_{\mu\nu}(\nu, Q^2)\epsilon^\nu(\lambda), \tag{17}$$

where $K = \sqrt{\nu^2 + Q^2}$ is the flux of virtual photons (using the Gilman convention [16]) and $\epsilon^\nu(\lambda)$ is the polarization vector of the virtual photon. Among these different $\sigma_\lambda^{\gamma^*N}$, only four cross sections are independent under rotational, parity and time reversal invariance. They are labeled by the photon polarization T (transverse) and L (longitudinal), and can be explicitly expressed in terms of the structure functions,

$$\sigma_{3/2}^T = \frac{4\pi^2\alpha}{K}[W_1(\nu, Q^2) - M\nu G_1(\nu, Q^2) + Q^2 G_2(\nu, Q^2)], \tag{18}$$

$$\sigma_{1/2}^T = \frac{4\pi^2\alpha}{K}[W_1(\nu, Q^2) + M\nu G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2)], \tag{19}$$

$$\sigma_{1/2}^L = \frac{4\pi^2\alpha}{K}[W_2(\nu, Q^2)(1 + \frac{\nu^2}{Q^2}) - W_1(\nu, Q^2)], \tag{20}$$

$$\sigma_{1/2}^{TL} = \frac{4\pi^2\alpha}{K}\sqrt{Q^2}[MG_1(\nu, Q^2) + \nu G_2(\nu, Q^2)]. \tag{21}$$

These four independent cross sections contain all the information about the hadron structure governed by the underlying strong interaction theory.

Combining Eqs. (18), (19) and (21) the spin dependent structure function $G_1(\nu, Q^2)$ can now be expressed in terms of three total absorption cross sections $\sigma_{1/2}^T$, $\sigma_{3/2}^T$, and $\sigma_{1/2}^{TL}$,

$$G_1(\nu, Q^2) = \frac{1}{8\pi^2 \alpha M} \frac{\nu}{\sqrt{\nu^2 + Q^2}} [\sigma_{1/2}^T - \sigma_{3/2}^T + \frac{2\sqrt{Q^2}}{\nu} \sigma_{1/2}^{TL}]. \tag{22}$$

To investigate the Q^2–dependence of the GDH sum rule, it is convenient to define the GDH integral [14,15]

$$I(Q^2) = M \int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu} G_1(\nu, Q^2). \tag{23}$$

It is clear that the $\sigma_{1/2}^{TL}$ term vanishes at $Q^2 = 0$ so the original GDH sum rule for a real photon can be recovered. In the perturbative region, this term also vanishes. Thus, $I(Q^2)$ is a generalization for the GDH sum rule suitable to study the Q^2-dependence of the sum rule.

III. HELICITY AMPLITUDES IN THE CONSTITUENT QUARK MODEL

As is well known, in the small Q^2 region the dominant mechanism for virtual photon absorption is through photo-excitation of baryon resonances. Therefore, the total photoabsorption cross sections, $\sigma_{1/2}^T$, $\sigma_{3/2}^T$ and $\sigma_{1/2}^{TL}$, can be calculated in the constituent quark model which has been quite successful in hadronic physics. In the following subsection, we shall express the total absorption cross sections in terms of helicity amplitudes $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$.

A. Helicity amplitudes and electromagnetic interaction

The total cross sections for the electromagnetic excitation of a given resonance from the nucleon can be completely specified by the corresponding helicity amplitudes $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$. As illustrated in Fig. 2. these helicity amplitudes are defined as follows

$$A_{1/2} = \langle {}^d[\underline{B}]_J, [\underline{A}, L^P]_N, S_z = 1/2| H_t^{em} |{}^2[\underline{8}]_{1/2}, [\underline{56}, 0^+]_0, S_z = -1/2 \rangle, \tag{24}$$

$$A_{3/2} = \langle {}^d[\underline{B}]_J, [\underline{A}, L^P]_N, S_z = 3/2| H_t^{em} |{}^2[\underline{8}]_{1/2}, [\underline{56}, 0^+]_0, S_z = 1/2 \rangle, \tag{25}$$

and

$$S_{1/2} = \langle {}^d[\underline{B}]_J, [\underline{A}, L^P]_N, S_z = 1/2 | H_l^{em} | {}^2[\underline{8}]_{1/2}, [\underline{56}, 0^+]_0, S_z = 1/2 \rangle, \tag{26}$$

where $|{}^d[\underline{B}]_J, [\underline{A}, L^P]_N, S_z\rangle$ stands for the spin-flavor-space part of the baryon wave function in the standard $[SU_{\mathrm{SF}}(6) \otimes O(3)]_{\mathrm{sym}} \otimes SU_c(3)$ classification scheme. The $[\underline{A}, L^P]_N$ are the $SU_{\mathrm{SF}}(6)$ multiplets with $\underline{A} = \underline{56}$, $\underline{70}$, or $\underline{20}$, L is the total orbital angular momentum, N the total quantum number of excitations of all the modes and $P = (-1)^N$ is the parity of the state. The ${}^d[\underline{B}]_J$ specifies the $SU_F(3)$ multiplet states, with $\underline{B}=\underline{1}$, $\underline{8}$, or $\underline{10}$ (singlet, octet, and decuplet), d the spin multiplicity and J the total angular momentum of the state. The proton in the initial state is thus denoted by $|{}^2[\underline{8}]_{1/2}, [\underline{56}, 0^+]_0, S_z\rangle$, with $S_z = \pm 1/2$.

The H_t^{em} and H_l^{em} are the transverse and longitudinal transition operators, respectively, and

$$H_t^{em} = H_{NR} + H_{SO} + H_{NA} \tag{27}$$

with

$$H_{NR} = -\sum_{i=1}^{3}[\frac{e_i}{2m_i}(\vec{p}_i \cdot \vec{A}_i + \vec{A}_i \cdot \vec{p}_i) + \mu_i \vec{\sigma}_i \cdot \vec{B}_i - e_i \phi_i], \tag{28}$$

$$H_{SO} = -\frac{1}{2}\sum_{i=1}^{3}\left[2\mu_i - \frac{e_i}{2m_i}\right]\frac{\vec{\sigma}_i}{2m_i}(\vec{E}_i \times \vec{p}_i - \vec{p}_i \times \vec{E}_i), \tag{29}$$

$$H_{NA} = \sum_{i<j}\frac{1}{4M_T}(\frac{\vec{\sigma}_i}{m_i} - \frac{\vec{\sigma}_j}{m_j})(e_j\vec{E}_j \times \vec{p}_i - e_i\vec{E}_i \times \vec{p}_j), \tag{30}$$

where the subscript i refers to i-th quark, and the three components of the Hamiltonian correspond to the nonrelativistic (NR), spin-orbit (SO), and non-additive (NA) interactions, respectively. The longitudinal transition operator, H_l^{em}, is defined as

$$H_l^{em} = \epsilon_0 J_0 - \epsilon_3 J_3, \tag{31}$$

where $\epsilon_\mu = \frac{1}{\sqrt{Q^2}}\{k_3, 0, 0, \nu\}$ is the photon longitudinal polarization vector and J_μ the nucleon electromagnetic current. Using the gauge invariance condition $k_\mu J^\mu = k_\mu \epsilon^\mu = 0$, one obtains $\langle \psi_f | H_l^{em} | \psi_i \rangle = \frac{\sqrt{Q^2}}{k_3}\langle \psi_f | J_0 | \psi_i \rangle$. Therefore, the matrix element $\langle \psi_f | H_l^{em} | \psi_i \rangle$ does not depend on the choice of the current as long as it is gauge invariant. It is noted that the longitudinal transition is proportional to $\sqrt{Q^2}$ and so vanishes in the real photon limit. In

terms of quarks, the zeroth component of the longitudinal transition operator is expressed by

$$J_0^{em} = \frac{1}{\sqrt{2\nu}}\{\sum_j \left[e_j + \frac{ie_j}{4m_j^2}\vec{k}\cdot(\vec{\sigma}_j \times \vec{p}_j) \right] e^{i\vec{k}\cdot\vec{r}_j}$$
$$- \sum_{j<i} \frac{i}{4M_T}\left(\frac{\vec{\sigma}_j}{m_j} - \frac{\vec{\sigma}_i}{m_i}\right)[e_j(\vec{k}\times\vec{p}_i)e^{i\vec{k}\cdot\vec{r}_j} - e_i(\vec{k}\times\vec{p}_j)e^{i\vec{k}\cdot\vec{r}_i}]\}. \tag{32}$$

The M_T in Eq. (30) and (32) is the total mass of the quark system. The first term in Eq. (32) is the charge operator used in the conventional calculation of the longitudinal helicity amplitudes, while the second and third terms are the spin-orbit and non-additive terms which have counterparts in the transverse electromagnetic transition and represent the relativistic corrections to the first term of order $O(v^2/c^2)$. In Ref. [17] it was pointed out that the spin-orbit and non-additive terms are crucial in reproducing the experimental data. However, most studies have neglected these two terms. In this work, we include these two terms in our calculations. Furthermore, we note that the Hamiltonian used here satisfies the low energy theorem [9]. The details of our calculations for these helicity amplitudes, $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$, are given in the previous papers [18].

B. The total absorption cross sections

Given $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ for each resonance, the total virtual photon absorption cross sections, $\sigma_{1/2}^T$, $\sigma_{3/2}^T$, and $\sigma_{1/2}^{TL}$, can be easily obtained. Using a Breit-Wigner parameterization for each resonance, the total photoabsorption cross sections are the incoherent sums of the contributions from the individual baryon resonances [19],

$$\sigma_{1/2,3/2}^T(\nu, Q^2) = \sum_R \left(\frac{2M}{W+W_R}\right)\frac{\Gamma_R}{(W-W_R)^2 + \Gamma_R^2/4}\left|A_{1/2,3/2}^R(\nu, Q^2)\right|^2, \tag{33}$$

$$\sigma_{1/2}^{TL}(\nu, Q^2) = \sum_R \left(\frac{2M}{W+W_R}\right)\frac{\Gamma_R}{(W-W_R)^2 + \Gamma_R^2/4}[A_{1/2}^{R*}S_{1/2}^R + S_{1/2}^{R*}A_{1/2}^R] \tag{34}$$

where W is the center-of-mass energy, $W^2 = (p+q)^2$, W_R and Γ_R are the mass and the total width of the resonance R, and $A_{1/2,3/2}^R$ and $S_{1/2}^R$ correspond to the helicity amplitudes for the transverse and longitudinal photon to produce resonance R, respectively.

IV. RESULTS AND CONCLUDING REMARKS

The calculation of the helicity amplitudes $A^R_{1/2}$, $A^R_{3/2}$ and $S^R_{1/2}$ has been carried out for 17 baryon resonances: $P_{33}(1232)$, $P_{11}(1470)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$, $S_{31}(1620)$, $D_{15}(1675)$, $D_{33}(1675)$, $D_{13}(1720)$, $F_{15}(1680)$, $P_{13}(1720)$, $P_{31}(1920)$, $P_{35}(1905)$, $P_{33}(1920)$, $P_{37}(1950)$, $P_{11}(1705)$, and $P_{33}(1600)$. Non-relativistic harmonic oscillator wave functions, which have been shown to reproduce the well-known properties of these baryon resonances quite successfully, are used in the calculation. The helicity amplitudes in the constituent quark model are consistent with the experimental data for the first few low-lying resonances. For further details of the calculation, we refer to Ref. [18].

To compare with the experimental cross sections, the non-resonant background contributions to the same processes have to be considered. Here we use a phenomenological method which ignores possible interference between background and resonances in the same multipole channel [20]. Our predicted Q^2 dependence of the GDH sum rule is shown in Fig. 3. The solid curve represents the full contributions from all 17 resonances included, whereas the dash-dotted curve gives the $P_{33}(1232)$ contribution alone. Clearly, the bulk features of the Q^2 dependence come from the dominant $P_{33}(1232)$ resonance for small momentum transfers. However, the contributions from higher resonances are sizable especially after the sign-flip point ($Q^2 \sim 0.3$ GeV^2) where the $P_{33}(1232)$ contribution nearly vanishes. In the same figure, we also show the pQCD Q^2–evolution [see Eq. (2)] with the dashed line. There is a big gap in the small Q^2 region.

(i) The GDH sum rule is saturated by the contributions from baryon resonances at a confidence level of 94%. The predicted value of the GDH integral, $I(Q^2 = 0)$, is 0.492 GeV^{-2} which is to be compared with $\frac{2\pi^2\alpha}{M^2}\kappa^2 = 0.524$ GeV^{-2} for the proton.

(ii) The GDH sum rule has a strong Q^2-dependence below $Q^2 = 1.0$ GeV^2, and a sign flip at about $Q^2 = 0.3$ GeV^2. This is in agreement with the recent data from the E143 collaboration at SLAC which indicates that the sign flip point should be even smaller than 0.5 GeV^2. A weak Q^2 dependence is predicted in the region of Q^2 larger than 1.0 GeV^2.

This Q^2-dependence comes mainly from the contribution of the $P_{33}(1232)$ resonance. At small momentum transfers, the main portion (roughly 75%) of the GDH sum rule integral is provided by the contribution of $P_{33}(1232)$ photoproduction. This contribution is negative at small Q^2, turns positive around $Q^2 = 0.3 \text{ GeV}^2$ and soon decreases rapidly with a further increase of Q^2. Our interpretation is the following: at small Q^2, the magnetic dipole transitions dominate $P_{33}(1232)$ photoproduction and $\sigma_{1/2}/\sigma_{3/2} \sim 3$. At large Q^2, because of chiral invariance, only $\sigma_{1/2}$ survives and this leads to a sign change in the GDH integral. For larger Q^2, the form factor for the proton to $P_{33}(1232)$ transition starts to act and quickly makes the $P_{33}(1232)$ contribution negligible after $Q^2 \sim 1 \text{ GeV}^2$.

(iii) Our analysis also shows that the behavior in the deep inelastic region cannot be naively extended to small Q^2 region where the resonances dominate (see the dash-dotted line in Fig. 3). Qualitative agreement with the extrapolated EMC analysis at moderate Q^2 may be achieved only when the dominant contribution from $P_{33}(1232)$ resonance is neglected. One cannot use pQCD in the small Q^2 region.

(iv) The calculation also shows that the contribution from $\sigma_{1/2}^{TL}$ cannot be neglected in the small Q^2 region. The longitudinal transitions of baryon resonances play an important role in the understanding of baryon structure. In particular, the longitudinal transition amplitude $S_{1/2}(Q^2)$ decreases as Q^2 increases and it is therefore important to consider the $S_{1/2}(Q^2)$ contribution in the small Q^2 region, especially for the transitions between the nucleon and the P-wave resonances, which are accessible in the experiments at TJNAF.

In conclusion, we have tested the GDH sum rule numerically by studying inelastic electron-proton scattering in the constituent quark model. The virtual photon coupling to the nucleon was assumed to occur mainly through the excitation of baryon resonances where the helicity amplitudes could be calculated in the constituent quark model. The electromagnetic interaction Hamiltonian which we used includes spin-orbit and non-additive terms in addition to the usual nonrelativistic pieces. Thus the all important relativistic effects at medium and high energy were effectively included. Note that the interference cross

section, $\sigma_{1/2}^{TL}$, which has often been omitted before, is now included in our calculation.

Of course, there are many aspects of this calculation which could, in principle, be improved. For example, because of the tensor structure of the hyperfine interaction the total spin and the total orbital angular momentum are no longer separately conserved, and this will induce $SU(6)_{\text{SF}} \otimes O(3)$ configuration mixing. The corresponding baryon states are therefore superpositions of the $SU(6)_{\text{SF}} \otimes O(3)$ basis states,

$$|\Phi_{\text{baryon}}\rangle = \sum_i c_i |\Phi_{SU(6)_{\text{SF}} \otimes O(3)}^i\rangle. \tag{35}$$

This configuration mixing may change the prospective theoretical result. It is worthwhile to recall that there are two assumptions in deriving the GDH sum rule. The first is the low energy theorem for forward Compton scattering, which is well accepted, and the second is the validity of the unsubtracted dispersion relation [see Eq. (8)]. The GDH sum rule itself is general and model independent. It is reasonably supported by the pion photoproduction data [21] and should provide an important constraint and test for models which describe electromagnetic transitions of the nucleon.

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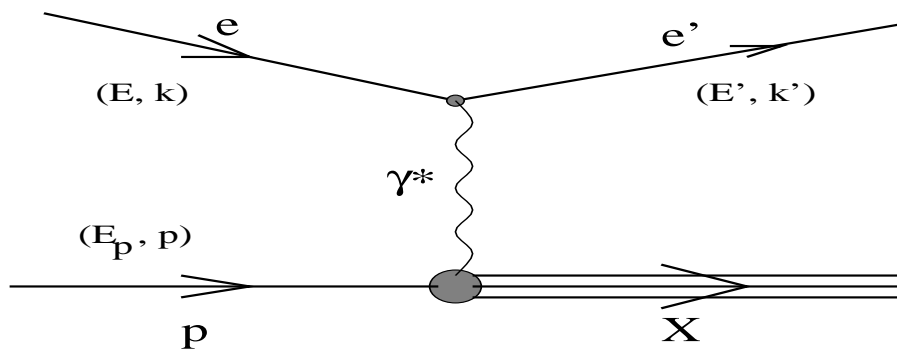
FIG. 1. Schematic representation of electron deep inelastic scattering from the nucleon. γ^* stands for a virtual photon.
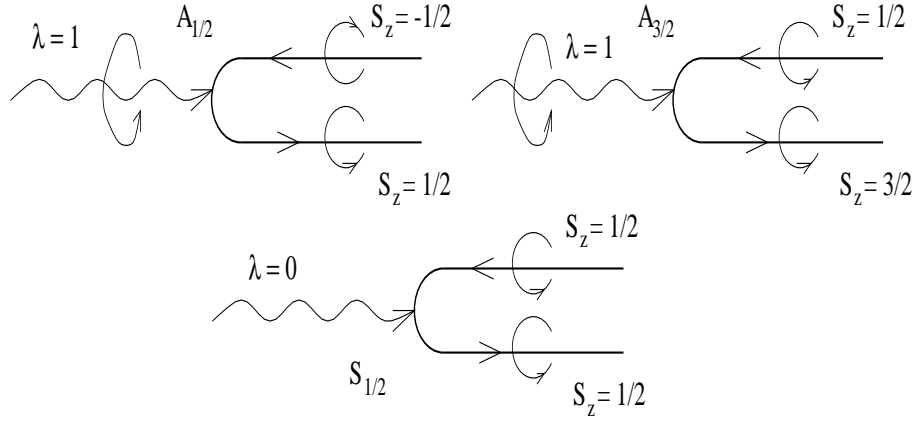
FIG. 2. Illustration of helicity amplitudes for a virtual photon scattering off a nucleon. The amplitudes $A_{1/2}$ and $A_{3/2}$ are for a transverse photon being absorbed by a nucleon with antiparallel and parallel spin projections with respect to the photon, and $S_{1/2}$ corresponds to a longitudinal photon.
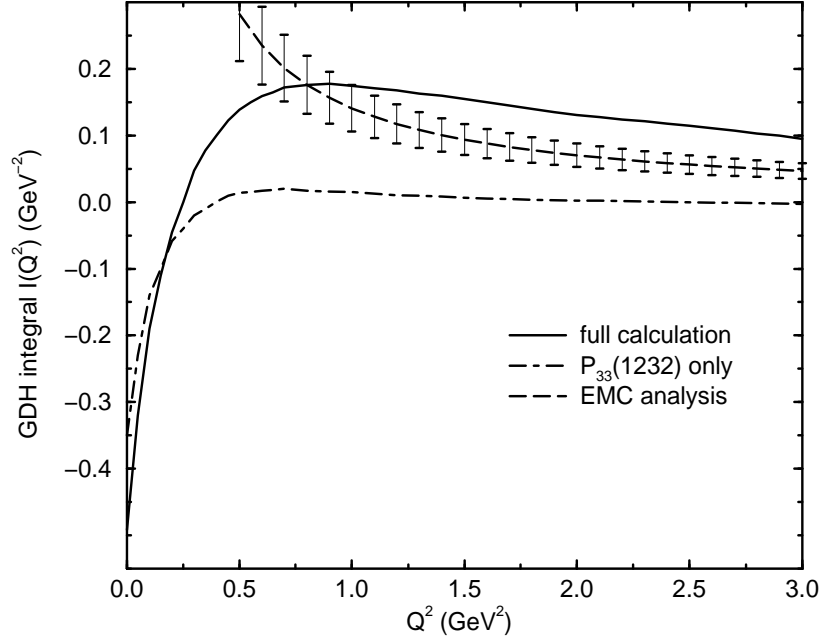
FIG. 3. Q^2-dependence of the GDH sum rule. The solid curve represents total contributions from 17 baryon resonances while the dot-dashed curve gives the $P_{33}(1232)$ contribution alone. The dashed line with error bars is from the pQCD analysis of the EMC experiment which is only valid for large momentum transfer.